OSCILLATORY REGIMES OF CONVECTION IN A TWO-LAYER SYSTEM

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The complete nonlinear equations are used to study the oscillatory convective motions arising in a system of two liquids with an interface.

Finite-amplitude stationary convective motions in a two-layer system, heated from below, were investigated in [1]. The stationary motions, however, do not exhaust the entire set of possible flow regimes. For example, in solving the problem of convection in water at a temperature near 4°C, oscillations were observed [2].

In this paper, based on the solutions of the complete nonlinear equations, we study the nonstationary convection in a system of two liquids with an interface. In a cavity with a prescribed wall ratio, transitions are realized with a change in the Grashof number between five flow regimes, differing by their spatial structure and their variation as a function of time. It is found that the region of stochastic oscillations is bounded with respect to the Grashof number from below as well as from above by regions with regular oscillations of different type.

We shall examine a cavity with a rectangular cross section, heated from below and filled by two different viscous immiscible liquids. We assume that the interface between the liquids is horizontal and is not subjected to deformation (the surface tension is large); the thermocapillary effect is not studied. All boundaries of the cavity are solid boundaries. The horizontal boundaries are held at different constant temperatures (the temperature difference equals θ), and the temperature varies linearly along the vertical boundaries. The origin of coordinates is positioned at the interface, the x axis is oriented horizontally, and the y axis is oriented vertically. The upper liquid fills the region $0<x<l,\ 0<y<a_1$, and the lower liquid fills the region $0<x<l,\ -a_2<y<0$.

The coefficients of dynamic and kinematic viscosity, thermal conductivity, temperature diffusivity, and volume expansion of the upper and lower liquids equal $\eta_i,\ \nu_i,\ \varkappa_i,\ \chi_i,\ \beta_i$ $(i=1,2)$, respectively.

We introduce the following notation: $\eta=\eta_1/\eta_2,\ \nu=\nu_1/\nu_2,\ \varkappa=\varkappa_1/\varkappa_2,\ \chi=\chi_1/\chi_2,\ \beta=\beta_1/\beta_2,\ L=l/a_1,\ a=a_2/a_1$. For the units of length, time, stream function, vortex velocity, and temperature, we select $a_1,\ a_1^2/\nu_1,\ \nu_1,\ \nu_1/a_1^2,\ \theta$, respectively. We shall write the complete nonlinear equations of free convection for the stream function ψ_i, the vortex velocity φ_i, and the temperature T_i in dimensionless variables:

$$\frac{\partial\varphi_i}{\partial t}+\frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_i}{\partial x}-\frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_i}{\partial y}=d_i\Delta\varphi_i+Gb_i\frac{\partial T_i}{\partial x},$$
$$\Delta\psi_i=-\varphi_i, \tag{1}$$
$$\frac{\partial T_i}{\partial t}+\frac{\partial\psi_i}{\partial y}\frac{\partial T_i}{\partial x}-\frac{\partial\psi_i}{\partial x}\frac{\partial T_i}{\partial y}=\frac{c_i}{P}\Delta T_i,$$

where $d_1=b_1=c_1=1;\ d_2=1/\nu;\ b_2=1/\beta;\ c_2=1/\chi;\ G=g\beta_1\theta a_1^3/\nu_1^2$ is the Grashof number; $P=\nu_1/\chi_1$ is the Prandtl number. The boundary conditions at the solid walls have the form:

$$x=0,\ x=L,\ \psi_i=\frac{\partial\psi_i}{\partial x}=0\quad(i=1,\ 2),$$

$$y=1,\ \psi_1=\frac{\partial\psi_1}{\partial y}=0,\ y=-a,\ \psi_2=\frac{\partial\psi_2}{\partial y}=0, \tag{2}$$

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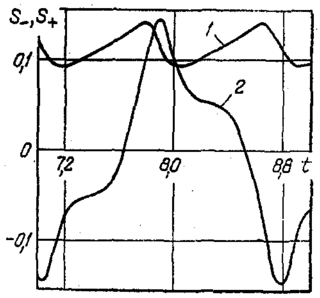

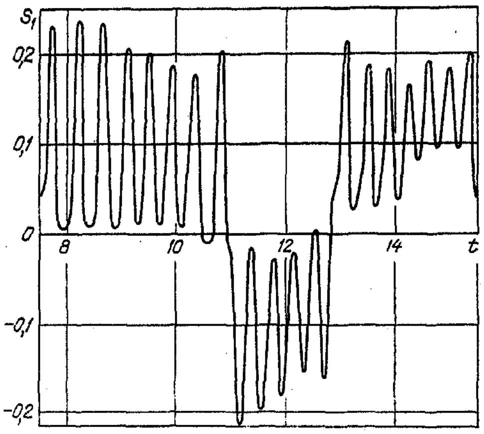

Fig. 1 Fig. 2

Fig. 1. Dependence of $S_-(1)$ and $S_+(2)$ on t.

Fig. 2. Dependence of S_1 on t (G = 9900).

$$y=-a,\ T_2=1,\ y=1,\ T_1=0,$$

$$x=0,\ x=L,\ T_1=\frac{1-y}{1+\varkappa a},\ T_2=\frac{1-\varkappa y}{1+\varkappa a}.$$

At the interface the normal components of the velocity vanish (nondeformable boundary), the tangential components of the velocity, the tangential components of the stresses, the temperatures, and the thermal fluxes are continuous:

$$y=0,\ \psi_1=\psi_2=0,\ \frac{\partial\psi_1}{\partial y}=\frac{\partial\psi_2}{\partial y},\ \eta\varphi_1=\varphi_2,$$

$$T_1=T_2,\ \varkappa\frac{\partial T_1}{\partial y}=\frac{\partial T_2}{\partial y}. \tag{3}$$

The boundary-value problem formulated above is determined by seven physical (G, P, η, ν, $\varkappa$, χ, β) and two geometrical (L, a) parameters.

System (1)-(3) was solved with the help of the method of finite differences. The computational procedure is described in detail in [1]. We used an explicit factorization method with central differences. The calculations were performed on a uniform 32 × 32 grid. Poisson's equation was solved by the method of iterations (Liebman's scheme with successive upper relaxation). The accuracy of the iteration of Poisson's equation constituted 10^{-7}. The velocity vortex at the solid walls was determined from Kuskova–Chudov's formula [3]. Conditions (3) were investigated to calculate φ_i and T at the interface:

$$\varphi_1(x,\ 0)=\frac{-2\left[\psi_2(x,\ -\Delta y)+\psi_1(x,\ \Delta y)\right]}{(\Delta y)^2(1+\eta)},$$

$$\varphi_2(x,\ 0)=\eta\varphi_1(x,\ 0),$$

$$T_1(x,\ 0)=T_2(x,\ 0)=\frac{T_2(x,\ -\Delta y)+\varkappa T_1(x,\ \Delta y)}{1+\varkappa}.$$

Here Δy is the step in the grid along the vertical coordinate. The magnitude of the step in time was selected from the conditions of stability of the calculation.

We shall describe the results obtained for the system water–silicon oil (Dow Corning N200) with the following set of parameters: $\eta=0.915$; $\nu=1.116$; $\varkappa=0.169$; $\chi=0.472$; $\beta=7.1595$; P = 6.28 at L = 0.8. For the characteristics of the flow structure, we introduce the quantities:

$$S_1=\int_0^{L/2}dx\int_0^1 dy\,\psi_1(x,\ y),\ S_2=\int_{L/2}^{L}dx\int_0^1 dy\,\psi_1(x,\ y), \tag{4}$$

$$S_+=S_1+S_2,\ S_-=S_1-S_2.$$

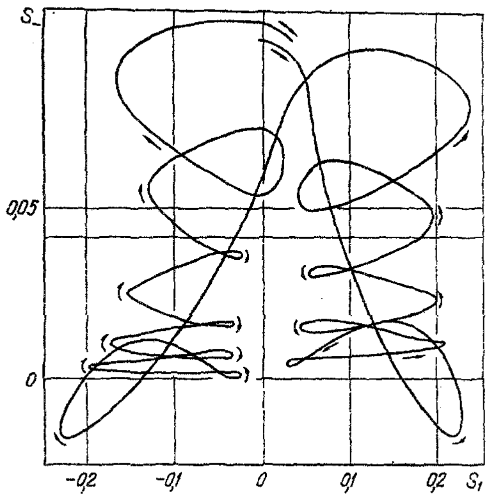

Fig. 3

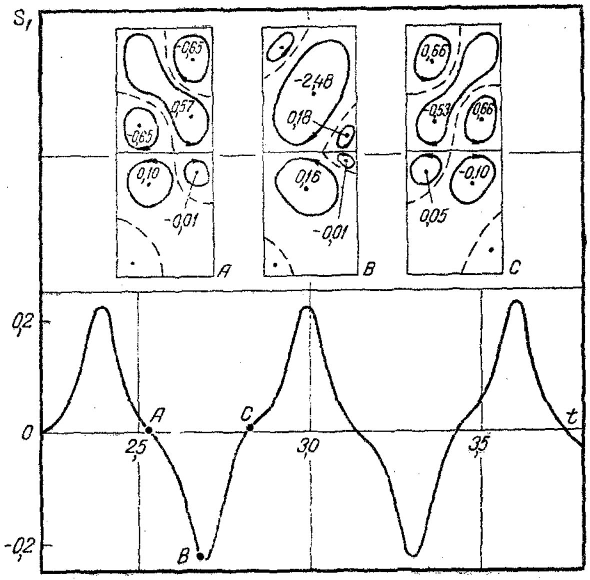

Fig. 4

Fig. 3. Phase trajectory.

Fig. 4. Dependence of S_1 on t (G = 11,000) and the patterns of isolines at the points A, B, and C.

For $G < G_0 = 1700$, mechanical equilibrium is conserved in the system. When the threshold Grashof number is exceeded, the equilibrium becomes unstable and a stationary convective motion develops (regime 1); in addition, the intensity of the motion in the upper liquid is much higher than in the lower liquid. This can be explained as follows. We introduce Rayleigh's numbers, determined for the upper and lower liquids (see [1]):

$$R_1 = \frac{GP}{\varkappa + 1}, \quad R_2 = R_1 \frac{\nu\chi\varkappa}{\beta}.$$

For the selected values of the parameters, the ratio $R_1/R_2 \simeq 10^2$. As a result of this condition, convection in the upper liquid is attained at much lower values of G than in the lower liquid. For this reason, intense convective motion due to volume (buoyancy) force develops only in the upper liquid, while only a weak flow, caused by the tangential stresses at the interface, exists in the lower liquid. We note that for stationary motion the difference between the quantities S_1 and S_2, due to the difference in the boundary conditions at the horizontal boundaries of the upper layer, is small ($|S_1 - S_2|/S_1 < 0{,}1$); the motion in the upper liquid is primarily a single-vortex motion.

As G increases, the motion becomes unstable and regular oscillations develop in the system (regime 2). The structure of the motion in the upper liquid changes considerably: the motion becomes a two-vortex motion and, in addition, the intensity of the vortices varies periodically with time. The quantity S_- (curve 1 in Fig. 1; G = 5700), characterizing the contribution of the two-vortex component of the motion, is not small and its sign is conserved during the oscillations; the contribution of the single-vortex mode S_+ (curve 2 in Fig. 1) is sign-alternating. Hysteresis occurs between regimes 1 and 2. In the region G > 6700 the oscillations become irregular (regime 3); the double-vortex flow structure remains. For higher values of G, the motion becomes a superposition of the single- and four-vortex structure (regime 4). A fragment of the typical pattern of the change in the quantity S_1 with time is presented in Fig. 2. Figure 3 shows the phase trajectory for the given case. The number of oscillations in the regions $S_1 > 0$ and $S_1 < 0$ varies irregularly.

For $G > G_* = 10{,}025 \pm 5$, the oscillations again become regular (regime 5; see Fig. 4). Thus, in the problem under examination, the region of irregular oscillations is bounded with respect to the parameter G both from below and from above. Figure 4 also shows the pattern of isolines corresponding to points A, B, and C. The oscillations retain their regular character in the entire region of values of the Grashof number investigated (right up to G =

25,000). As G increases, the period of the oscillations decreases from 0.89 to 0.24 as G varies from 10,030 to 20,000.

NOTATION

x and y, Cartesian coordinates; ψ, stream function; φ, velocity vortex; T, temperature; η, coefficient of dynamic viscosity; ν, coefficient of kinematic viscosity; $\varkappa$, coefficient of thermal conductivity; β, coefficient of volume expansion; θ, temperature difference; G, Grashof's number; P, Prandtl's number; L and a, geometric parameters; χ, coefficient of thermal diffusivity. The indices 1 and 2 refer to the upper and lower liquids, respectively.

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APPLICATION OF THE PROJECTION-NET METHOD FOR SOLVING THE TRANSIENT HEAT-TRANSFER PROBLEM IN AN ANNULAR DUCT OF COMPLEX CONFIGURATION

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The influence of the geometrical characteristics of ducts on various parameters of the heat-transfer processes taking place in them is investigated.

A topic of practical importance in the study of heat conduction and convective heat transfer is the influence of the geometrical characteristics of the investigated objects, ducts in particular, on various parameters of the processes involved [1]. To study the dependence of the temperature field on the geometrical characteristics and to obtain a realistic picture of the heat-transfer processes in a duct it is necessary to investigate simultaneously the processes of heat conduction in the wall and heat transfer in the fluid, i.e., to solve the problem in the conjugate formulation [2].

Analytical methods for the solution of conjugate transient (time-dependent) convective heat-transfer problems have not been adequately developed [2, 3], and their application is rendered difficult by the need to allow for the cross-sectional geometry and the boundary conditions specified on the outer surface of the wall. In our opinion, therefore, the projection-net method is the most promising approach to the solution of the indicated problems; it combines the finite-element method (FEM) with the finite-difference method (FDM).

The inherent capability of using irregular nets in the FEM permits the curvilinear boundaries of the computational domain to be effectively approximated, and the variational formulation of the problem makes it easy to take various types of boundary conditions into account. Another advantage of the FEM is the feasibility of forming the system of equations automatically; this is achieved by inspecting each element separately and applying a conditioning procedure that will ensure continuity of the function at the interelement boundaries. The FDM ensures the necessary speed and accuracy of the computations in analyzing the behavior of the heat transfer with time and in the direction of motion of the fluid.

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